Exploiting Arc Splines for Digital Maps

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Abstract— **In modern driver assistance systems digital maps have proven usefulness for many applications in providing information on the environment around the host vehicle. For future ADAS, relevant to traffic safety, high precision digital maps including detailed information about the course of individual lanes are needed.**

We introduce a new method for the generation of highly accurate digital maps including lane specific information based on a wide range of raw data sources. We show that the use of arc splines as a representation model of individual lanes is advantageous for both computational efficiency and accuracy. Therefore, this method is useful for a wide variety of applications using digital maps to enrich driving comfort and traffic safety.

I. INTRODUCTION

Today's advanced driver assistance systems (ADAS) benefit from precise and up-to-date digital maps of the environment around the host vehicle. For map-based self-localization strategies, where map features are associated with sensor measurements in order to derive the vehicle's position, high precision digital maps including information on the individual lanes are needed. Safety systems like curve speed control applications also ask for high-accuracy lane maps when extracting information on the upcoming curve to determine the maximum speed at which the vehicle may drive safely. Furthermore, situation analysis and map-matching tasks, which imply the association of objects to lanes, require detailed road course information next to the ego-vehicle. Finally, separated lane information could enrich driving comfort applications like navigation systems.

The amount of data and its level of detail, stored in digital map databases, are enriched permanently. High precision localization hardware like RTK-GPS enables the registration of the surroundings by using different sensors. The captured data must be processed in order to find an efficient representation of the relevant environment structures.

A. Motivation

According to the German road construction act (cf. [1], [2]) many turns in rural areas are constructed using clothoidal parts between line and arc segments to provide a smooth steering phase when passing the lane section. This is achieved by the property of the clothoid curve, whose curvature varies linearly with its arc length. However, clothoids show certain disadvantageous properties, which will be discussed later on.

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In general, a representation of the lane is preferable, that shows invariance to offset determination, rotation, translation and scaling. Basically, all aforementioned applications have in common that, for a given point, the calculation of a best approximating point on a lane must be calculated. This is necessary for issues like map matching, map-based self-localization and the extraction of curve information on some lane. Due to the high frequency of this computation appearing in real-time driver assistance systems, the calculation must be as efficient as possible.

In this work we focus on the representation of lanes in digital maps. Analogously, representations for road markings, curbstones and guardrails can be calculated using the same approach. More precisely we discuss different curve models and finally propose an approach for the generation of high accuracy road maps using arc splines.

B. Related Work

The most simple but commonly used representation for lanes in digital maps is based on polygons (cf. [3], [4], [5]). An overview of different methods on the generation of road course maps from raw data points is presented in [6]. In [7] a road refinement based on an active shape model is proposed in order to fit B-splines to road segments. Walton and Meek [8] discuss a control polygon approach for guiding a clothoid spline. Many research studies deal with clothoid approximations in order to get around the transcendental structure of the clothoid. In [9] a strategy is pointed out on the approximation of a clothoid and its offset by Bézier curves and B-splines. By the use of an arc spline approximation to a clothoid it is shown in [10] that the approximation error decreases by a quadratic order when increasing the number of arcs.

C. Outline

Starting with the data acquisition in section II-A, we discuss different curve type as possible lane representations in section II-B taking into account a predefined list of relevant criteria. Next, a new method for the generation of an arc spline representation is pointed out in part III. After showing some experimental results in section III-C, we conclude our work with some future ideas.

II. METHOD DESCRIPTION

A. Data Acquisition

For the generation of a curve representation of the lane, we assume to have a set of data points to be approximated according to a chosen model. These data points can arise from processing raw data of different data sources like

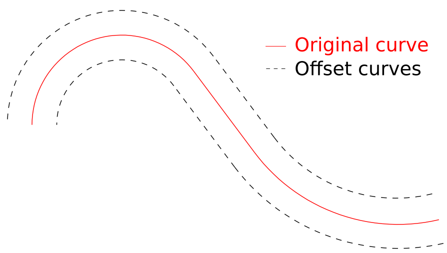
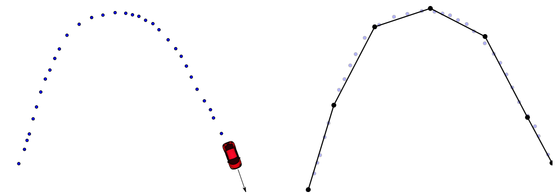


Fig. 1: Illustration of an offset curve



(a) Bird's eye view on data points for subsequent curve fitting.

(b) Illustration of a polygon representation.

Fig. 2: Example for a simple lane representation.

- laser scanner-based data acquisition
- video-based lane detection
- processing of aerial images
- import from other data sources like maps or reference measurements.

B. *Discussion of Lane Representations*

1) Evaluation Criteria: In this section we discuss different lane representations that are commonly used for digital maps. Beside their general properties we focus on the following criteria, which are important for the applications:

Precision: In order to evaluate the precision of the chosen lane representation we must define the ground truth reference data and the error metric. For that purpose it is reasonable to refer to an abstract set of point data as ground truth, as it can be extracted from existing reference maps or other ground truth sources. Well-established error metrics are based on the maximum norm or least square error.

Calculation of best approximating points: As mentioned in the introduction the calculation of best approximating points on the curve with respect to a given point is a crucial issue. Therefore, high efficiency is needed in order to enable real-time algorithms on the applications side. Hence, any non-linear optimization for that task should be avoided during the running driver assistance system.

Offset curve: Some elements like road markings or guardrails are located parallel to the lane in terms of offset curves (cf. Fig. 1). Hence the calculability of an offset curve is an important criterion.

Data volume: The data volume for the lane representation should be as memory efficient as possible to ensure data handling on the applications side.

2) Curve Model: As mentioned above we assume to have a set of data points, that should be approximated by the chosen lane model. In Fig. 2a a simple example is given for illustration.

a) Polygon: A very simple, but commonly used model for the lane representation is given by a polygon, approximating the data points (cf. Fig. 2b). For cartographic purposes it can be defined manually using an interactive tool for the map generation or using approximation techniques like the *minimum link path* (MLP) (cf. [11]). Polygons do not provide any curvature information which is indeed important for many applications. Furthermore, they only satisfy continuity, but no smoothness (C^1-continuity) at the breakpoints which is counterintuitive as roads are actually constructed with smooth course. In section III-C some statements are given concerning the minimum necessary number of polygon breakpoints for a road segment and a predefined precision. This estimation directly defines the minimum data amount that is necessary to store the polygon representation. The calculation of point to curve distances is simple for a polygon and can be expressed in a closed form. The offset curve of a polygon is an arc spline.

b) Clothoids: A clothoid is a curve given in a parametric form by the Fresnel integral. For instance, the clothoid at the origin with starting curvature zero is defined by

$$K : [0, L] \to \mathbb{R}^2, K(t) := a \begin{pmatrix} C(t/a) \\ S(t/a) \end{pmatrix},$$

$$C(t) := \int_0^t \cos\left(\frac{\pi}{2} s^2\right) ds, \quad S(t) := \int_0^t \sin\left(\frac{\pi}{2} s^2\right) ds,$$

$$a = \sqrt{\pi L R},$$

where L is the arc length, R is the curvature radius at the end point of the clothoid and a is a scaling factor. The curvature of the clothoid increases linearly in terms of the arc length. As mentioned above, clothoids are used for certain parts of road constructions. Therefore, for these parts, a clothoid would be the best model for our map building task. However, most of the relevant calculations, like curve approximation, point to curve distance calculation and even drawing of the curve are computationally expensive as they imply non-linear optimizations. In general, the offset curve of a clothoid is not any more a clothoid. Regarding the data amount, at least the start point and the starting tangential direction have to be stored together with two values of either the arc length, the end curvature or the scaling factor.

c) Polynomial splines: There are several works proposing cubic Hermit splines ([6]), B-Splines ([7]) or NURBS ([9]) as an approximation of clothoids. Most of these approaches are based on interpolation between breakpoints, that have to be defined beforehand either manually or by a strategy using original data points. However, this choice is subjective but crucial as it is sensitive to outliers. While some of the polynomial splines guarantee C^2-continuity, the calculation of point to curve distances still remains a non-linear optimization problem. Furthermore, invariance criteria with respect to rotation, translation, scaling and offset building are only hardly satisfied or comparatively expensive

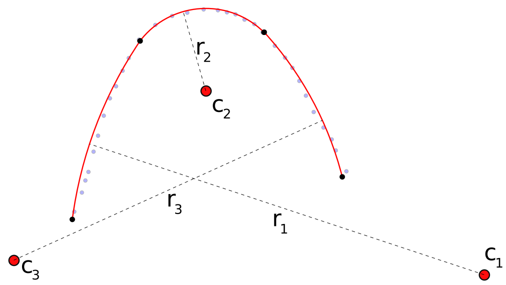


Fig. 3: Curve approximation using an arc spline

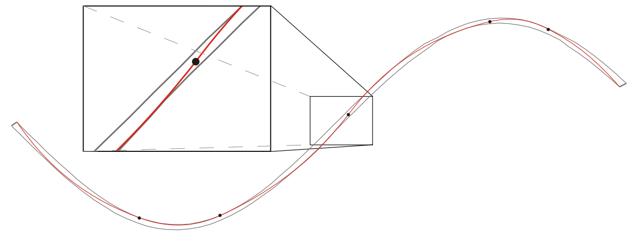


Fig. 4: Example of a tolerance channel and a corresponding SMAP. The small points indicate the breakpoints of the arc spline. Also, we can see that the pictured SMAP alternately touches the channel from the left and from the right.

to compute. For each curve segment, simply the parameter of the polynomials have to be stored.

d) Arc splines: An arc spline is a planar curve, whose trace is a union of circular arcs and line segments, and which must be capable of being injectively parameterized. The latter means that the curve has no self-intersections. Accordingly, *smooth* arc splines are those ones which show equality of the tangent unit vectors at the breakpoints given by every two corresponding segments. Further properties of arc splines are their curvature being a step function and the invariance with respect to rotations, translations and scalings. Beyond that, the offset curve of an arc spline is in fact an arc spline and can be calculated very easily, and the calculation of point to curve distances is especially simple as it can be expressed in a closed form. In contrast to clothoids or polynomial splines, an arc spline can always be expressed in a parameter-free form. In addition, they are compatible with all established geometry and CAD systems. Visualization of an arc spline is computationally much more efficient in comparison to other curve types mentioned above. A circular arc is uniquely defined by three distinct points. Hence concerning the amount of data for storage, for each segment at most two points must be stored if the breakpoints are reused for the next segment and additionally one more point for the whole arc spline. Section III-C provides information on the number of segments, that are necessary for the curve approximation.

Another advantage of using arc splines is that effectively round structures like roundabouts can be represented especially well in digital maps. Finally, if an application needs a more simple representation, a polygon can be extracted directly by connecting the breakpoints of the arc spline.

Due to their advantageous properties, we propose to take smooth arc splines as a model for the representation of lanes and similar structures in digital maps. As the map generation should preferably run without any user interaction, the question remains how to calculate the arc splines based on a set of data points. A simple exemplary situation is illustrated in Fig. 3, where a smooth arc spline is fitted to the above example.

III. Generation of Arc Splines

A. Smooth Minimum Arc Path (SMAP)

We are interested in a curve that not only approximates the extracted points up to unavoidable tolerance errors but also describes them effectively, i.e. with minimal complexity. Such a characterization allows coping with tasks and applications motivated in Section I. In addition to their general properties, *(circular) arc splines* satisfy the evaluation criteria for modeling lanes formulated in Section II-B. Above all, smoothness at the breakpoints is required in order to enable a realistic modeling.

A promising solution would be a smooth arc spline approximating the data points with respect to a given tolerance. This approach turns the approximation problem outlined above into a multi-objective optimization: Obviously, the approximation error diminishes if the number of line and arc segments increases. The more exactly the points are approximated the more segments are needed. Hence the proposed method minimizes the number of segments while keeping a given tolerance. Additionally, this tolerance can possibly be adjusted to vary locally if desired.

Our approach controls the approximation error by only focusing on solutions staying inside a so-called *tolerance channel*. Typically, such a channel is given by a simple polygon or an arc spline (cf. Fig. 4). In addition, a source and a destination segment are fixed. Any smooth arc spline staying inside the tolerance channel and connecting the source and destination segments with a minimum number of segments solves the problem. Such a spline is called *smooth minimum arc path (SMAP)*.

In [12] an efficient algorithm based on 'alternating sequences' and 'feasible direction sets' for generating a SMAP is presented. Alternating sequences are families of points on the bounding curve of the channel that are alternately touched from the left and from the right as indicated in Fig. 4. Though generating a SMAP guarantees a smooth arc spline with the minimally possible number of segments with respect to any accuracy, it doesn't satisfy real time requirements. However, the worst case complexity is quadratic with respect to the number of input points N and the SMAP algorithm even performs in $O(Nk)$ in the most practical applications, where k is the number of segments. Apart from that, the computational time doesn't play an important role since the lane approximation can be generated off-line.

Note that this algorithm simply needs a set of points and a desired accuracy as input data. Then, a corresponding tolerance channel is calculated and a SMAP is generated. In comparison, the method suggested in [10] would need the original clothoid, arc and line segments of the lane, which have to be approximated. However, usually only point data, not exact reference curves are available for lane approximation. Finally, the question remains how to generate a suitable tolerance channel from some data in order to use the algorithm proposed in [12].

B. Tolerance Channels

After a preprocessing step, where we possibly have removed some outliers, we can suppose a finite family of points $(p_1, \ldots, p_N)$ with $P := \{p_1, \ldots, p_N\}$ s.t. the polygonal curve ω successively passing through the points p_i, starting at p_1 and ending at p_N is simple. As already indicated, we address the approximation of P by a (smooth) arc spline with a minimal number of segments subject to satisfying the bounding requirements of an error function. Hence we are searching for a smooth arc spline γ with a minimum number of segments subject to $\Phi(\gamma, P) < \varepsilon$ for some $\varepsilon > 0$ and error function Φ. Dealing with the maximum norm

$$\Phi(\gamma, P) = \max_{i=1,\ldots N} \operatorname{dist}(p_i, \gamma),$$

would be appropriate, where dist denotes the euclidean distance. Alternatively, the Hausdorff-distance of ω can be taken into account. For algorithmic causes it is reasonable to design a suitable tolerance channel and to use the methods proposed in [12]. This approach has a considerable advantage. We are able to control the behavior of the approximating curve between each two points p_i and p_{i+1} by the bounding channel. This way we also get geometric constraints, which can be locally varied in quite a simple manner and are easier to modify than constraints defined by a metric or norm.

When computing an approximation s.t. the determined arc spline is within a specified tolerance $\varepsilon > 0$ to the p_i, the offset $\Omega_\varepsilon(\omega) = \{a \in \mathbb{R}^2 \mid \operatorname{dist}(a, \omega) = \varepsilon\}$ of the polygonal path ω can be considered. As already mentioned, the offset curve of a polygonal is an arc spline for sufficiently small ε. In order to ensure an efficient algorithmic approach we can again approximate the arcs in the offset by a polygonal path, and we obtain a polygon channel, where mostly the semi-circles at p_1 and p_N are replaced by a start and a destination line segment. Generally speaking, the ε-offset is a region formed from strips of width 2ε which are centered at the polygon edges. Thus, in a neighborhood of sharp corners this doesn't guarantee that the curve remains close to the given points. Therefore, Drysdale et al. suggest a so-called *polygonal tolerance region* in [13]. They also want their approximating curve to have distance at most $\varepsilon > 0$ from ω. Fig. 6 shows an example of a polygonal tolerance region, which ensures the Hausdorff distance of γ and ω to be smaller than ε for every curve γ from the starting to the destination segment staying inside the closure of this region. Although the channel depicted in Fig. 5 might not exactly guarantee a Hausdorff distance less or equal ε, it is appropriate if P doesn't yield sharp corners. Furthermore, it can be computed straightforwardly and is sufficient for most applications with real data.

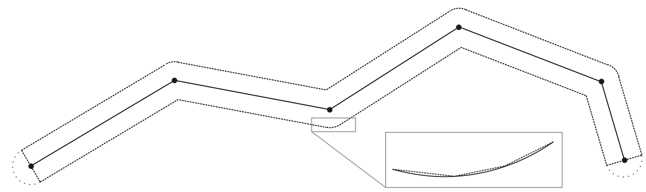


Fig. 5: Polygonal Path connecting points and corresponding offset curve for some $\varepsilon > 0$; start and destination segments are line segments passing through the first and last point, respectively.

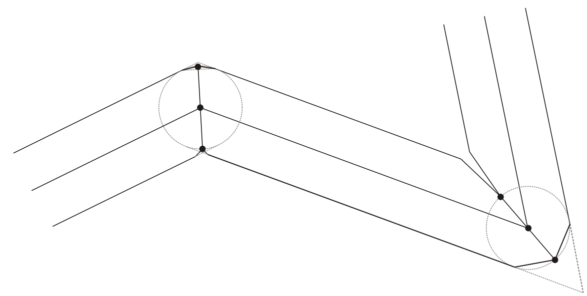


Fig. 6: Polygonal tolerance region. The bends at the two points are shortcut in order to guarantee the approximating curve having at most distance ε, which is indicated by the gray circles with radius ε.

If we have some additional preliminary knowledge of some parts the approximating curve should consist of, we can flexibly adapt the tolerance channel passing through this part. For instance, knowing that in a certain neighborhood there should be a line but not an arc segment, we can search for a line connecting corresponding arcs or the other way around, i.e. we first fit a line through a subset of the given points and then force the arc spline corresponding to the remaining points to join the line smoothly. For instance, the approximation illustrated in Fig. 9 was established this way. In addition, it is possible to integrate preliminary knowledge by inserting vertices automatically or manually without losing the minimal number of segments.

C. Experimental Results

Figures 7 and 8 show examples of the approximation of a clothoid with maximally admissible tangent deviation given by an arc length $L = r$, where r is the curvature radius at the end point and the starting curvature vanishes.

From the theoretical results of [10] and [12] we can state the following properties:

- If the clothoid, which has to be approximated, has a constant arc length then the number of arcs increases when increasing the tangent deviation angle θ. As θ

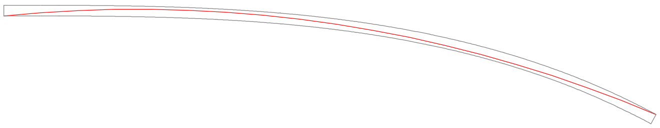


Fig. 7: Tolerance channel of a clothoid with $L = R = 30$ meters and width 2ε, $\varepsilon = 0.2$ meters.

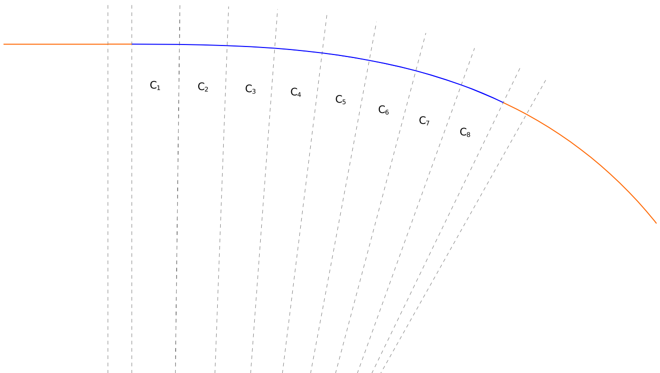


Fig. 8: Tolerance channel of a clothoid with $L = R = 1000$ meters and width 2ε, $\varepsilon = 0.1$ meters; exemplary arc spline approximation with 8 segments smoothly joining a given line and arc segment.

is bounded by a maximally feasible angle due to [1], we can estimate the maximum number of segments a SMAP needs at most for an arbitrary tolerance level.

- Narrowing the tolerance channel to a n-th of the previous width meets in scaling the original clothoid with factor n while keeping the tolerance error fixed. That means that the ratio of arc length and maximum error stays constant.
- Using n-times the number of segments the approximation error behaves like $O(n^{-2})$.
- If L is the maximally possible clothoid arc length such that only one arc is sufficient for approximating it with tolerance ε, we also need only one arc when approximating a clothoid with n-fold arc length and tolerance $\varepsilon \cdot n$.

In our tests we have focused on the following scenario: We have approximated a clothoid which joins a starting line segment and an ending arc with radius r in a C^2-manner. Table I shows the maximal arc length such that one arc suffices for the approximation of a clothoid with $L = r$ and $L = \frac{r}{9}$ respectively. As already seen, the case $L = r$ corresponds to the maximum tangent deviation and $L = \frac{r}{9}$ to the minimum one. Some examples of estimations of the number of segments needed regarding a tolerance error of $\varepsilon = 0.05,\ 0.1,\ 0.2$ meters can be found in Tables II to IV. There we have compared approximations by a smooth arc spline with corresponding polygonal curves given by a MLP.

Note that the illustrated values are upper bounds of the real amount of segments a SMAP needs since the method we used for calculating these values doesn't guarantee the minimally possible number of segments. In contrast, the number of line segments is a lower bound, as they were created by calculating a MLP, which ensures the minimality with respect to a polygonal tolerance channel. We will adapt and optimize a SMAP-based strategy for map generation purposes within the project Ko-PER. In any case, we usually need a fewer number of arc segments since supposing a maximum tangent deviation is an extremal setting regarding all feasible clothoids and this doesn't meet the average case.

Fig. 9 shows an example of a lane approximation based on raw data. The line segments at the beginning and at the end have been approximated beforehand and then a SMAP smoothly joining these line segments has been computed. Note that the width of the lane varies, i.e. the width at curved parts is greater than at straight ones. Hence the resulting arc splines are not parallel curves. Also, for the right curve we have plotted the corresponding tolerance channel and we included a magnified detail showing the extracted raw points the tolerance channel was generated from.

	$\varepsilon = 0.05$m	$\varepsilon = 0.1$m	$\varepsilon = 0.2$m
Maximum tangent deviation	8.43m	16.86m	33.73m
Minimum tangent deviation	74.77m	149.56m	299.12m

TABLE I: Estimations for maximum arc length such that one arc is sufficient for approximation.

	15	25	50	100	250	500	1000	2500	5000
I	2	2	3	4	6	8	11	18	25
II	3	4	6	8	12	17	24	38	53
III	1	1	1	2	2	3	4	6	9
IV	2	2	2	3	5	6	8	13	18

TABLE II: Estimations for the number of segments w.r.t. to tolerance $\varepsilon = 0.05$ m and different arc lengths L (meters). I Arc Spline with $L = r$, II Polygon with $L = r$, III Arc Spline with $L = r/9$, IV Polygon with $L = r/9$.

	15	25	50	100	250	500	1000	2500	5000
I	1	2	2	3	4	6	8	13	18
II	3	3	4	6	9	12	17	27	38
III	1	1	1	1	2	2	3	5	6
IV	1	1	2	2	3	5	6	9	13

TABLE III: Estimations for the number of segments w.r.t. to tolerance $\varepsilon = 0.1$ m and different arc lengths L (meters). I Arc Spline with $L = r$, II Polygon with $L = r$, III Arc Spline with $L = r/9$, IV Polygon with $L = r/9$.

	15	25	50	100	250	500	1000	2500	5000
I	1	1	2	2	3	4	6	9	13
II	2	2	3	4	6	9	12	19	27
III	1	1	1	1	1	2	2	3	5
IV	1	1	1	2	3	3	5	7	9

TABLE IV: Estimations for the number of segments w.r.t. to tolerance $\varepsilon = 0.2$ m and different arc lengths L (meters). I Arc Spline with $L = r$, II Polygon with $L = r$, III Arc Spline with $L = r/9$, IV Polygon with $L = r/9$.

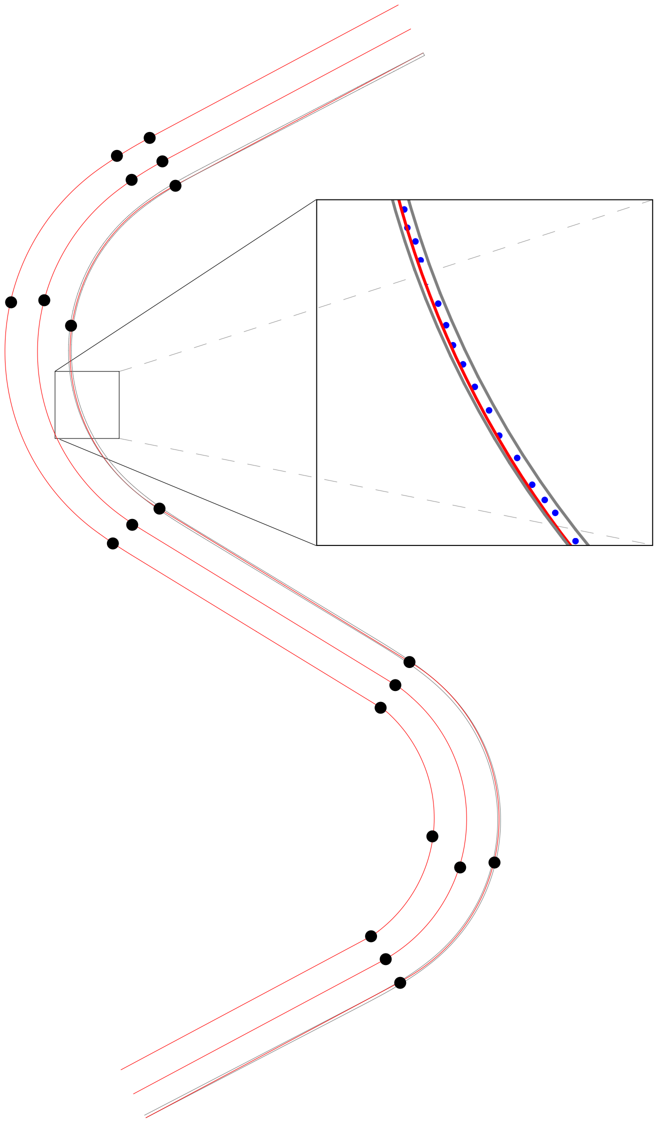


Fig. 9: Approximation of a S shaped lane by smooth arc splines. The left and the middle splines have eight segments and the right one consists of seven arcs and line segments.

IV. CONCLUSIONS AND FUTURE WORKS

In this paper we presented an approach for the usage of arc splines as curve representation in high precise digital maps. We demonstrated that arc splines show advantageous properties for computational efficiency which is necessary in many driver assistance systems that use digital maps. Furthermore, for cartographic purposes, an efficient method has been pointed out that allows the generation of arc splines based on a wide range of data sources taking into account a freely adjustable approximation accuracy.

Several extensions of our approach will be developed in future: We will enrich our lane representation with compact and dynamic information on the lane width and also height profiles will be addressed. First evaluations give reason to yielding an efficient and high accurate 3D representation when fusing the planar curve and the height information.

V. ACKNOWLEDGMENTS

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